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14. ABSTRACT

This collaborative project aims at the application and development of state-of-the-art theory and computational methods in optimization and game theory for the dynamic resource allocation problem. This optimization problem is practically important due to its central role in dynamic spectrum management and economic planning. We have investigated centralized and decentralized versions of the dynamic resource management problem, including the nonconvex system maximization problem of users' achievable utilities, non-cooperative models of user behavior under budget constraints, pricing models of cognitive radio systems, optimal design of spectral masks and interference scale factors, the presence of malicious players, and distributed algorithms and their convergence analysis for the practical solution of the models. We have developed new methodologies that not only deal directly with these applied problems, but also fundamentally enhance the frontier of mathematical programming and game theory.

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Final Report

Collaborative AFOSR grant: FA9550-09-1-0329 A3004
University of Illinois at Urbana-Champaign

Project Period: from May 01, 2009 to November 30, 2011

Project Title: Optimization Algorithms and Equilibrium Analysis for Dynamic Resource Allocation

Principal Investigator: Jong-Shi Pang (University of Illinois at Urbana-Champaign)

Synopsis: This collaborative project aims at the application and development of state-of-the-art theory and computational methods in optimization and game theory for the dynamic resource allocation problem. This optimization problem is practically important due to its central role in dynamic spectrum management and economic planning. We have investigated centralized and decentralized versions of the dynamic resource management problem, including the nonconvex system maximization problem of users' achievable utilities, non-cooperative models of user behavior under budget constraints, pricing models of cognitive radio systems, optimal design of spectral masks and interference scale factors, the presence of malicious players, and distributed algorithms and their convergence analysis for the practical solution of the models. We have developed new methodologies that not only deal directly with these applied problems, but also fundamentally enhance the frontier of mathematical programming and game theory.

Publications: The publications that resulted from this grant are summarized as follows.

• Nonconvex games with side constraints, *SIAM Journal on Optimization* 21 (2011) 1491–1522.

Abstract. This paper develops an optimization-based theory for the existence and uniqueness of equilibria of a non-cooperative game wherein the selfish players' optimization problems are nonconvex and there are side constraints and associated price clearance to be satisfied by the equilibria. A new concept of equilibrium for such a nonconvex game, which we term a "quasi-Nash equilibrium" (QNE), is introduced as a solution of the variational inequality (VI) obtained by aggregating the first-order optimality conditions of the players' problems while retaining the convex constraints (if any) in the defining set of the VI. Under a second-order sufficiency condition from nonlinear programming, a quasi-Nash equilibrium becomes a local Nash equilibrium of the game. Uniqueness of a QNE is established using a degree-theoretic proof. Under a key boundedness property of the Karush-Kuhn-Tucker multipliers of the nonconvex constraints and the positive definiteness of the Hessians of the players' Lagrangian functions, we establish the single-valuedness of the players' best-response maps, from which the existence of a Nash equilibrium (NE) of the nonconvex game follows. We also present a distributed algorithm for computing a NE of such a game and provide a matrix-theoretic condition for the convergence of the algorithm. An application that pertains to a special multi-leader-follower game wherein the nonconvexity is due to the followers' equilibrium conditions in the leaders' optimization problems is presented. Another application to a cognitive radio paradigm in a signal processing game is described in two separate papers.

• M. RAZAVIYAYN, Z.Q. LUO, P. TSENG, AND J.S. PANG. A stackelberg game approach to distributed spectrum management. *Mathematical Programming, Series B* 129 (2011) 197-224.

Abstract. We consider a cognitive radio system with one primary (licensed) user and multiple secondary (unlicensed) users. Given the interference temperature constraint, the secondary users compete for the available spectrum to fulfill their own communication need. Borrowing the concept of price from market theory, we develop a decentralized Stackelberg game formulation for power allocation. In this scheme, the primary user (leader) announces prices for the available tones such that a system utility is maximized. Using the announced prices, secondary users (followers) compete for the available bandwidth to maximize their own utilities. We show that this Stackelberg game is polynomial time solvable under certain channel conditions. When the individual power constraints of secondary users are inactive (due to strict interference temperature constraint), the proposed distributed power controlmethod is decomposable across the tones and unlike normal water-filling it respects the interference temperature constraints of the primary user. When individual power constraints are active, we propose a distributed approach that solves the problem under an aggregate interference temperature constraint.Moreover, we propose a dual decomposition based power control method and show that it solves the Stackelberg game asymptotically when the number of tones becomes large.

• F. FACCHINEI, J.S. PANG, G. SCUTARI, AND LORENZO LAMPARIELLO. Monotone VI-constrained hemivariational inequalities: Distributed algorithms and power control in ad-hoc networks. *Mathematical Programming, Series A*, in revision (original submitted July 2011)

Abstract. We consider centralized and distributed algorithms for the numerical solution of a hemivariational inequality (HVI) where the feasible set is given by the intersection of a closed convex set with the solution set of a lower-level monotone variational inequality (VI). The algorithms consist of a main loop wherein a sequence of one-level, strongly monotone HVIs are solved that involve the penalization of the non-VI constraint and a combination of proximal and Tikhonov regularization to handle the lower-level VI constraints. Minimization problems, possibly with nonconvex objective functions, over implicitly defined VI constraints are discussed in detail. The methods developed in the paper are then used to successfully solve a new power control problem in ad-hoc networks.

• J.S. PANG AND G. SCUTARI. Joint sensing and power allocation in nonconvex cognitive radio games: Quasi-Nash equilibria. *IEEE Transactions on Signal Processing*, in revision.

Abstract. In this paper, we propose a novel Nash problem for Cognitive Radio (CR) networks composed by an arbitrary number of primary users (PUs) and secondary users (SUs) wherein each SU (player) competes against the others to maximize his own opportunistic throughput by choosing jointly the sensing duration, the detection thresholds, and the vector power allocation over a multichannel link. The formulation contains constraints on the transmit power (and possibly spectral masks) as well as the maximum individual/aggregate (probabilistic) average interference tolerable from the PUs. To keep the optimization as decentralized as possible, global interference constraints are imposed via pricing. The resulting players optimization problems are nonconvex and there are price clearance conditions associated to the nonconvex global interference constraints to be satisfied by the equilibria of the game, which make the analysis of the proposed game a challenging task; none of classical results in the game theory literature can be successfully applied. To deal with the nonconvexity of the game, we introduce new relaxed equilibrium concepts, the Quasi-Nash Equilibrium (QNE) and the Local Nash Equilibrium (LNE), and study their main properties, connection, and performance. Quite interestingly, the proposed game theoretical formulation yields a considerable performance improvement with respect to current centralized and decentralized

designs of CR systems, which validates the new concept of QNE.

• G. SCUTARI AND J.S. PANG. Joint sensing and power allocation in nonconvex cognitive radio games: Nash equilibria and distributed computation, *IEEE Transactions on Information Theory*, in revision (original submitted April 2011).

Abstract. In this paper, we propose a novel Nash problem for cognitive radio networks modeled as a Gaussian frequency-selective interference channel, wherein each secondary user (player) aims to maximize his own opportunistic throughput by choosing jointly the sensing duration, the detection thresholds, and the vector power allocation, subject to transmit power (and possibly spectral masks) constraints as well as individual/aggregate (probabilistic) average interference constraints imposed by the primary users. The resulting players optimization problems are nonconvex and there are bi-convex, thus nonconvex, side (i.e., coupling) constraints to be satisfied by the equilibrium of the game. The lack of convexity and boundedness of the players optimization problems, and the presence of price clearing conditions associated with the side constraints make the analysis of the proposed games a challenging task; none of classical results in the game theory literature can be successfully applied, even only to prove the existence of a solution of the game. The main contribution of this paper is to develop a novel optimization-based theory for studying the proposed nonconvex games; we provide a comprehensive analysis of the existence and uniqueness of a standard Nash equilibrium, develop distributed best-response based algorithms, and establish their convergence. The implementation of the proposed algorithms can be distributed among the players and has been experimented to converge in a few iterations, making them appealing in many practical cognitive radio scenarios.

Research in Progress: The PI continues to engage in active research in the area of competitive resource allocation. Together with a doctoral student supported by this grant, he is completing a joint paper on the maximum attainment system sum rate in the dynamic spectrum management problem under Gaussian interference channels with unbounded budgets. Another paper in progress deals with a competitive non-cooperative game in which each selfish player solves a minimax optimization problem; the challenge of the latter problem is how to design provably convergent distributed algorithms in the presence of nondifferentiability in the objective functions of the players' optimization problems.

Award: EURASIP 2011 Best Paper Award for the joint paper with Zhi-Quan Luo: *Analysis of iterative waterfilling algorithm for multiuser power control in digital subscriber lines* published in the Journal on Applied Signal Processing

Student Supervision: The grant has provided partial support of several Ph.D. students in their thesis research, including Alberth Alvarado and Dane Schiro. These two students are expected to complete their dissertation work in 2014 and 2013, respectively. The grant has also provided some travel subsidies to Drs. Francisco Facchinei and Gesualdo Scutari for their trips to visit the PI to carry out joint research on various aspects of the project.

Presentations: The PI has presented the results of this research at various conferences and institutions as described below.

• 17th International Conference on Digital Signal Processing (Corfu, Greece; July 2011)

• International School of Mathematics "Guido Stampacchia Workshop on Nonlinear Optimization,

Variational Inequalities and Equilibrium Problems (Erice, Italy; July 2010)

• NetEcon11, 6th Workshop on the Economics of Networks, Systems, and Computation (San Jose, California; June 2011)

• The International Workshop on Large-Scale Optimization Fudan University (Shanghai, China; May 2010)